

Fractional control of an industrial furnace

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ABSTRACT. The requirements of high production allied with product quality, process safety and environmental regulation, lead control systems to play a key role in the operation of chemical and biochemical plants. In petrochemical plants, furnaces are essential equipments for process operation and due to energy costs, adequate operation and control are of extreme importance for process economics. The search for new and more efficient control laws led to the development of fractional PID control algorithm, which is based on the use of fractional differential equations. In this work, a previously identified mathematical model of an actual industrial furnace is used for fractional PID control studies. Feedback loop in servo control was analyzed, focusing on the study of the influence of the controller parameters over control loop performance. Particularly, P, fractional PI and fractional PD controller were considered in this study. Simulations were carried out showing that the fractional controllers were able to perform set-point transitions. The control loop performance was evaluated by ITAE and ISE criteria, showing that, in this study, fractional PI is the best algorithm.

Key words: process control, fractional differential equation, fractional control, furnace.

RESUMO. Controle fracionário de um forno industrial. A necessidade da elevada produtividade aliada à qualidade dos produtos, segurança dos processos e a legislações ambientais levaram sistemas de controle a possuírem papel fundamental na operação de plantas químicas e petroquímicas. Em plantas petroquímicas, fornos são equipamentos fundamentais para a operação do processo e, pelo custo da energia, a operação e o controle adequados são de essencial importância para a economia do processo. A procura de novas e mais eficientes leis de controle levou ao desenvolvimento do algoritmo de controle PID fracionário, o qual é baseado no uso de equações diferenciais de ordem fracionária. Neste trabalho, um modelo matemático previamente identificado para um forno real é utilizado para estudos de controle fracionário. Foi analisado o problema de controle *servo* em malha retroalimentada (*feedback*), focando o estudo da influência dos parâmetros do controlador sobre o comportamento da malha de controle. Especificamente, controladores tipo P, PI fracionário e PD fracionário foram considerados neste estudo. Simulações foram realizadas mostrando que controladores foram capazes de fazer a transição de *setpoint*. O desempenho das malhas de controle foi avaliado com os critérios ITAE e ISE, mostrando que, neste estudo, o controlador PI fracionário foi o melhor algoritmo.

Palavras-chave: controle de processos, equação diferencial fracionária, controle fracionário, forno.

Introduction

Due to the needs of high production allied with product quality, process safety and environmental regulation, control systems play a key role in chemical and biochemical plants operation (SANTOS et al., 2005). Literature reports different conventional control algorithms, which have been successfully applied to the control of industrial furnaces (SEBORG et al., 2003).

Furnaces represent an important category of industrial equipment not only because of their

function of pre-heating chemical stream but also because of energy consumption issues. As a consequence, adequate performance of an industrial furnace is important for process economical performance (SMITH, 1995). Literature reports different applications of furnace modeling and control, ranging from lab to industrial scale. Neitzel and Lenzi (2001) report the control of a lab scale furnace modeled with a first-order integer model. Radhakrishnan and Mohamed (2000) report the use of neural networks for modeling purposes, however, the demand of experimental data for obtained a

trained and validated neural network is high, which may not be available in every scenario. Liao et al. (2009) present the use of fuzzy sets and logic for furnace identification and control. Vodros and Kollar (2003) report the identification of a furnace using frequency domain based models. Finally, Branar (2005) shows the importance of using heuristic rules for furnace modeling.

Fractional differential order equations represent a fast growing research field nowadays (OLDHAM; SPANIER, 2006). Different applications of fractional calculus have been reported ranging from diffusion studies (LENZI et al., 2009, 2010) to stock market applications (BIANCO; GRIGOLINI, 2007). Further details regarding the formalism of fractional calculus (HILFER, 2000) and the physical interpretation of fractional derivatives (MACHADO, 2003) are beyond the scope of this work and can be found elsewhere. Fractional control has been successfully applied to mechanical (POMMIER et al., 2002) and eletromechanical systems (SABATIER et al., 2004). To our knowledge, this work reports the first application of fractional control to industrial equipment. This work reports an alternative innovative tool for the identification of an industrial furnace, which was validated with experimental data reported in the literature (PODLUBNY, 1999).

The aim of this work is the study of fractional control of an industrial furnace. Fractional controllers of the PID family were used in feedback loop for servo control. Particularly, P, fractional PD and fractional PI controller were used to control the temperature of the leaving gas stream of furnace, which is described by a previously identified fractional model.

Theoretical framework

A fractional derivate can be obtained by several approaches. However, in this work, only the Caputo representation (CAPUTO, 1967), presented bellow, will be considered.

$${}_a D_x^\beta f(x) = \frac{1}{\Gamma(m-\beta)} \cdot \left[\int_a^x \frac{f^{(m)}(\tau)}{(x-\tau)^{\beta+1-m}} d\tau \right], \quad (1)$$

$$m \leq \beta \leq m+1; \beta \in \mathfrak{R}; m \in \mathfrak{N}$$

The first advantage of this representation is the fact that initial conditions of the fractional differential equations can be expressed in terms of integer order derivatives, which usually have a physical interpretation. Secondly, for this representation, the fractional derivate of a constant function is zero allowing the use of deviation variables (SEBORG et al., 2003), which considerably simplify the mathematical analysis when using Laplace transform because the initial conditions become zero. The feedback loop for servo control is illustrated by Figure 1.

The loop transfer function is given by Equation. (2).

$$\frac{Y(s)}{Y_{\text{SET-POINT}}(s)} = \frac{G_{\text{CONTROLLER}}(s)}{1 + G_{\text{CONTROLLER}} \cdot G_{\text{ACTUATOR}}(s) \cdot G_{\text{SENSOR}}(s) \cdot G_{\text{PROCESS}}(s)} \quad (2)$$

In this work, both actuator and sensor are assumed to have no dynamics, consequently,

$$G_{\text{ACTUATOR}}(s) = G_{\text{SENSOR}}(s) = 1 \quad (3)$$

The process transfer function can be obtained from fractional identification techniques and is given by Equation (4).

$$G_{\text{PROCESS}}(s) = \frac{1}{a \cdot s^\alpha + b} \quad (4)$$

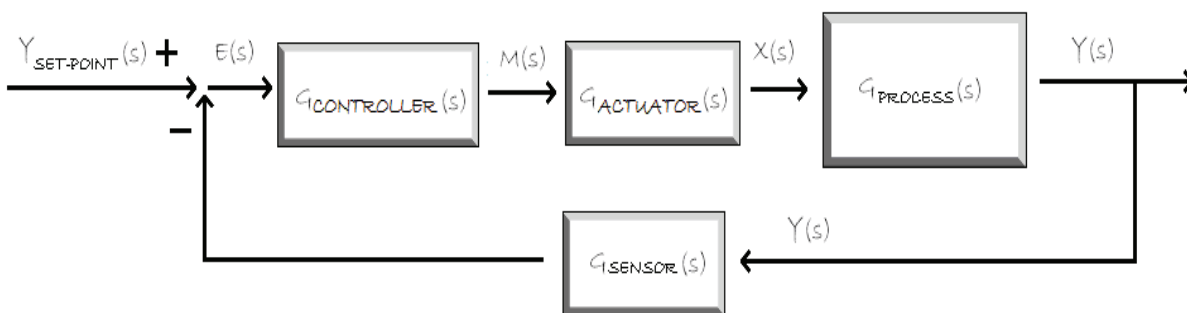


Figure 1. Feedback loop for servo control.

Finally, the P, fractional PD and fractional PI controllers are, respectively, given by the following transfer functions

$$G_{\text{CONTROLLER}}(s) = K_c \tag{5}$$

$$G_{\text{CONTROLLER}}(s) = K_c \cdot (1 + \tau_D \cdot s^\mu) \tag{6}$$

$$G_{\text{CONTROLLER}}(s) = K_c \cdot \left(1 + \frac{1}{\tau_I \cdot s^\lambda} \right) \tag{7}$$

Material and methods

The furnace model is given by Equation (8), which was identified from an actual industrial furnace data reported in the literature, using the identification technique reported by Isfer et al. (2010).

$$G_{\text{PROCESS}}(s) = \frac{1}{6.2992 \cdot s^{1.0252} + 1.8148} \tag{8}$$

This work addressed the study of fractional control loops, particularly focusing on the influence of the parameters of the control loop transfer function over the behavior of the controlled variable. Considering the above equations, the following control loop transfer functions can be obtained for P controller, fractional PI and fractional PD controller, respectively.

$$\frac{Y(s)}{Y_{\text{SET-POINT}}(s)} = \frac{G_{\text{CONTROLLER}}(s) \cdot G_{\text{PROCESS}}(s)}{1 + G_{\text{CONTROLLER}}(s) \cdot G_{\text{PROCESS}}(s)} = \frac{K_c}{6.2992 \cdot s^{1.0252} + [1.8148 + K_c]} \tag{9}$$

$$\frac{Y(s)}{Y_{\text{SET-POINT}}(s)} = \frac{K_c \cdot \tau_I \cdot (1 + s^\lambda)}{\tau_I \cdot 6.2992 \cdot s^{1.0252 + \lambda} + \tau_I \cdot [1.8148 + K_c] \cdot s^\lambda + K_c} \tag{10}$$

$$\frac{Y(s)}{Y_{\text{SET-POINT}}(s)} = \frac{K_c \cdot (1 + \tau_D \cdot s^\mu)}{6.2992 \cdot s^{1.0252} + K_c \cdot \tau_D \cdot s^\mu + [1.8148 + K_c]} \tag{11}$$

One can observe that parameter μ is a real number which can be set greater than 1.0252, so the transfer function needs to be rewritten as

$$\frac{Y(s)}{Y_{\text{SET-POINT}}(s)} = \frac{K_c \cdot (1 + \tau_D \cdot s^\mu)}{K_c \cdot \tau_D \cdot s^\mu + 6.2992 \cdot s^{1.0252} + [1.8148 + K_c]} \tag{12}$$

As mentioned before, only the servo control problem will be considered. Setpoint changes considered in this study consist of a unity step change, which is mathematically given by the Heaviside function.

Results and discussion

According to previously reported results (PODLUBNY, 1999), fractional controllers applied to processes described by fractional transfer functions (fractional process) provide better results when compared to integer controllers applied either to integer processes or even fractional processes and fractional controllers applied to integer processes. Consequently, this study analyzes only fractional controllers applied to fractional processes. P controller simulation is a benchmark for comparison.

P Controller

The inverse Laplace transform of the P controller loop is given by Equation (13), which described the behavior of the controlled variable.

$$y(t) = \left[\frac{K_c}{6.2992} \right] \cdot \epsilon_0 \left(t; -\frac{[1.8148 + K_c]}{6.2992}; 1.0252; 2.0252 \right) \tag{13}$$

This inverse transform is obtained with the aid of Mittag-Leffler function given by Equation (14).

$$\epsilon_k(t, x; \alpha, \beta) = t^{\alpha+k+\beta-1} \cdot E_{\alpha, \beta}^{(k)}(x \cdot t^\alpha) \tag{14}$$

$$E_{\alpha, \beta}^{(k)}(x \cdot t^\alpha) = \sum_{j=0}^{\infty} \frac{(j+k) \cdot (x \cdot t^\alpha)^j}{j! \cdot \Gamma(\alpha \cdot j + \alpha \cdot k + \beta)}$$

Table 1 presents the values of K_c used for simulation of the control loop. It must be stressed that the values were chosen in order to evaluate the influence of K_c over the behavior.

The behavior of the controlled variable after a unity step set-point change is shown in Figure 2. It can be observed that the offset is present; however, it can be reduced by increasing the value of K_c . The

independent variable, time, represents a dimensionless time. The presence of offset occurs because the P controller does not change the order of the differential equation that describes the process, as it can be observed by comparing Equation (8) to Equation (9). In this case, the fractional order is close to 1, so offset is expected to occur. However, care should be taken on the choice of κ_c , because the higher the value, the higher the process sensitivity to small errors.

Table 1. P controller parameter value used for simulation.

Parameter	Case			
	I	II	III	IV
κ_c	0.5	1	10	15

Fractional PI controller

The main advantage of PI controller is the offset elimination. This happens due to the integral action, which mathematically leads to an increase in the order of the differential equation that describes the process. This can be observed by comparison of Equation (8) to Equation (10). On the other hand, actuator saturation may be an important issue to be addressed when dealing with PI controllers. The inverse Laplace transform of Eq. (10) is given by Eq. (15).

$$y(t) = \left[\frac{\kappa_c}{e.2992} \right] \cdot \left(\sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \cdot \left(\frac{\kappa_c}{\tau_i \cdot e.2992} \right)^k \right) \cdot \left(\varepsilon \left(t, -\frac{\tau_i \cdot [\kappa_c + 1.8148]}{e.2992}, 1.0252, 1.0252 + \lambda - \lambda \cdot k + 1 \right) + \varepsilon \left(t, -\frac{\tau_i \cdot [\kappa_c + 1.8148]}{e.2992}, 1.0252, 1.0252 - \lambda \cdot k + 1 \right) \right) \quad (15)$$

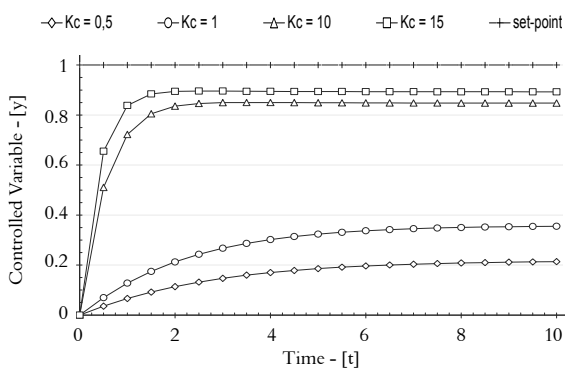


Figure 2. Controlled variable behavior – P Controller.

One can observe that fractional PI controller has three tuning parameters, given by κ_c , τ_i and λ . It should be stressed that the value of λ represents the desired increase of the order of the control loop differential equation and is given by a real number. Table 2 lists the parameters values used for simulation purposes.

Table 2. Fractional PI controller parameters values used for simulation.

Parameter	Case			
	I	II	III	IV
κ_c	5	5	5	1
τ_i	2	1	1	1
λ	0.1	0.1	1.1	1.1

Similarly to P controller, parameters values were chosen in order to evaluate the influence over the control loop behavior. The influence of κ_c is analyzed by comparison of cases III and IV, τ_i is analyzed by cases I and II, finally, λ is analyzed by cases II and III. Figure 3 presents the simulation results showing the behavior of the controlled variable. The main consequences of increasing κ_c correspond to an offset reduction and also an increase in the loop velocity by reducing the time needed to get closer to the desired set point. τ_i directly influences the offset, which can be removed by proper parameter selection. Finally, depending on the values of κ_c and τ_i , parameter λ may introduce an oscillatory behavior, as it represents the integral action and directly affects the order of the differential equation that describes the control loop.

Fractional PD controller

According to Equation (11) and Equation (12), fractional PD controller has three tuning parameters; κ_c , τ_D , μ . Two different scenarios can be visualized and will be further considered for analysis. In the first, the value of μ is smaller than the order of the transfer function that describes the process. In the second scenario, the value of μ is greater than the process order.

In the first scenario, given by Equation (11), the fractional PD response is expected to have the same behavior of the P controller as the order of the system is not changed. The main advantage of the PD controller is that the controlled variable reaches the set-point faster when compared to P controller. The control loop behavior is given by Equation (16), while Table 3 lists the values of the parameters used for simulations.

Figure 4 presents the controlled variable behavior, according to the used parameter values. Again, tuning parameters were not optimized. Cases I and VI and cases III and IV address the influence of κ_c . The influence of τ_D is evaluated by cases II and III, cases IV and V, cases VI and VII. Finally, parameter μ is studied by comparing cases I and II, cases V and VI, cases IV and VII. One can observe

that by increasing the values of K_c and τ_D offset is reduced. By increasing μ controlled variable offset tends to increase. However, an initial higher speed of the control loop is observed due to the sharp increase of the values of the controlled variable.

$$y(t) = \left[\frac{K_c}{6.2992} \right] \cdot \left(\sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \cdot \left(\frac{1.8148 + K_c}{6.2992} \right)^k \right) \cdot \varepsilon_k \left(t; -\frac{[\tau_D \cdot K_c]}{6.2992}; 1.0252 - \mu; 1.0252 - \mu \cdot k + 1 \right) + \tau_D \cdot \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \cdot \left(\frac{1.8148 + K_c}{6.2992} \right)^k \cdot \varepsilon_k \left(t; -\frac{[\tau_D \cdot K_c]}{6.2992}; 1.0252 - \mu; 1.0252 - \mu \cdot k + 1 - \mu \right) \quad (16)$$

Table 3. Fractional PD controller parameters values used for simulation – Scenario 1.

Parameter	Case						
	I	II	III	IV	V	VI	VII
K_c	1.5	1.5	1.5	0.3	0.3	0.3	0.3
τ_D	2	2	5	5	2	2	5
μ	1.01	0.2	0.2	0.2	0.2	1.01	1.01

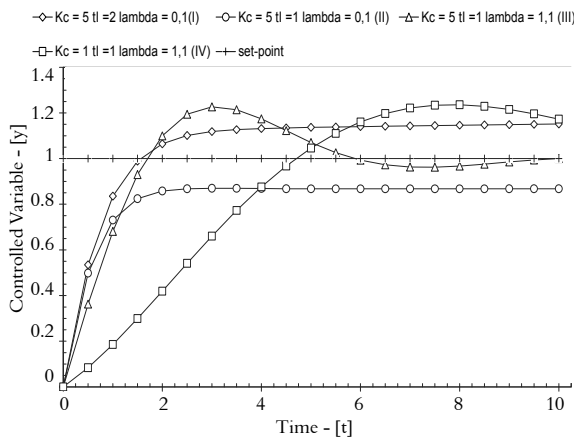


Figure 3. Controlled variable behavior – Fractional PI Controller.

In the second scenario, given by Equation (12), the controlled variable is described by the following expression obtained after inverse Laplace transform. Table 4 reports the used values of the control loop tuning parameters.

The influence of parameter K_c is observed from cases III and IV. The influence of τ_D is evaluated from cases II and III, and cases IV and VI. Finally, parameter μ is addressed by cases I and II, and cases IV and V. Figure 5 presents the simulations of the PD control loop behavior.

An increase in K_c and a decrease in τ_D lead to an offset reduction. Parameter μ plays a key role in the controlled variable behavior. This happens

because in scenario 2, μ influences the order of the closed loop transfer function, consequently, higher values of μ may introduce an oscillatory behavior. A proper parameter selection can considerably reduce offset not introducing oscillations.

$$y(t) = \left[\frac{1}{\tau_D} \right] \cdot \left(\sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \cdot \left(\frac{6.2992}{\tau_D \cdot K_c} \right)^k \right) \cdot \varepsilon_k \left(t; -\frac{[1.8148 + K_c]}{\tau_D \cdot K_c}; \mu - 1.0252; \mu - 1.0252k + 1 \right) + \tau_D \cdot \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \cdot \left(\frac{6.2992}{\tau_D \cdot K_c} \right)^k \cdot \varepsilon_k \left(t; -\frac{[1.8148 + K_c]}{\tau_D \cdot K_c}; \mu - 1.0252; \mu - 1.0252k + 1 - \mu \right) \quad (17)$$

Table 4. Fractional PD controller parameters values used for simulation – Scenario 2.

Parameter	Case					
	I	II	III	IV	V	VI
K_c	5	5	5	20	20	20
τ_D	10	10	5	5	5	1.2
μ	1.03	1.21	1.21	1.21	2	1.21

Performance evaluation

The control loop performance was evaluated using ITAE and ISE criteria, given by the following expressions, whose values are presented in Table 5.

$$ITAE = \int_0^t t \cdot |e(t)| dt = \int_0^t t \cdot |y(t) - y_{SET-POINT}| dt \quad (18)$$

$$ISE = \int_0^t [e(t)]^2 dt = \int_0^t [y(t) - y_{SET-POINT}]^2 dt \quad (19)$$

Table 5. Control loop performance criteria.

Controller	ITAE	ISE
P	5.41	0.42
Fractional PD – Scenario 1 (I)	12.06	0.93
Fractional PD – Scenario 2 (II)	4.53	0.32
Fractional PI	3.01	0.61

Considering the chosen tuning parameter values, fractional PI controller is the best if one considers ITAE, while fractional PD is the best if one considers ISE. It can be observed from the values of Table 5 and from Figure 6 that fractional control can be successfully used for servo control tasks.

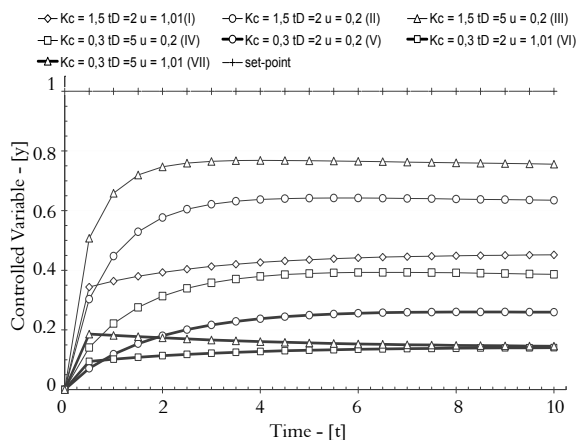


Figure 4. Controlled variable behavior – Fractional PD Controller – Scenario 1.

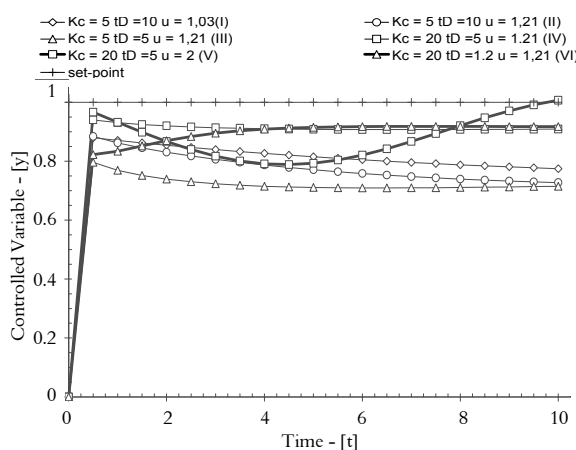


Figure 5. Controlled variable behavior – Fractional PD Controller – Scenario 2.

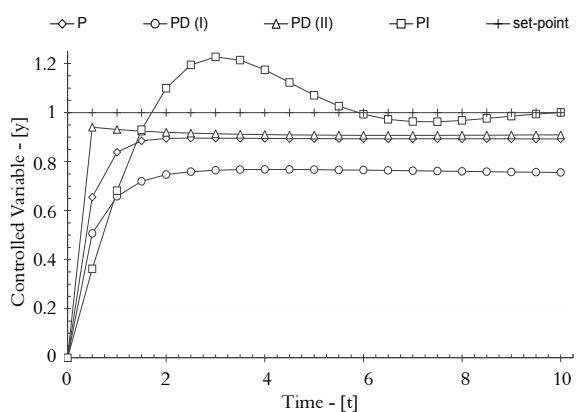


Figure 6. Comparison of the best controllers.

Conclusion

This work addressed the study of fractional control loop of an industrial furnace. P, fractional PD and fractional PI controllers were considered in this study. A fractional model of an industrial furnace previously reported in the literature was used for loop simulation. The influence of the

tuning parameters was analyzed showing that offset and oscillatory behavior can be eliminated by proper parameter selection. Although the parameters used were not optimized, they are able to allow the controlled variable to reach a new steady state close to the desired set-point. Finally, the control loop performance was evaluated by ITAE and ISE criteria, showing that each criterion has a different best controller.

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